1.Rationally biased rounding can be beneficial.

The larger gaps between  representable values in floating  slash
representation occur only  where one of  the boundary values  is
a particularly simple rational. Rounding by the number theoretic
concept of  best  rational approximation  appropriately  chooses
the simple rational. For problems  where simple rational  values
may occur and be meaningful,· this feature could be very  useful.
E.g.linear  programming and  combinatorial optimization are  two
of many areas where simple rational output may often arise.
    Two references  providing more   information  on this  feature
are:

> Matula,  D.W. and  Kornerup,  P.: "Approximate Rational
> Airithmetic  Systems: Analysis of  Recovery  of Simple
> Fractions During  Expression Evaluation",  Symbolic and
> Algebraic Computation,  E.W. Ng, ed.,  Lecture Notes in
> Computer  Science 72,  Springer-Verlag,  Berlin,  1979,
> 383-397.
> Matula,  D.W.  and  Ferguson,  W.: "Rationally  Biased
> Arithmetic", Proc. 7th  Sym. on Comp.  Arith., IEEE Cat
> #85CH2146-9, 1985,  194-202.

2.A  precision  fill  feature can  be  added to  floating  slash
representation.
The   standard   separate   numerator   and   denominator   word
representation of  rational  fractions provides  limitations  in
both range and· precision  features for many desired  approximate
real applications. Thus for approximation  of reals in  floating
slash we  must consider  both a  range problem  and a  precision
problem.
    Range  problem:    Our previously  described  extended  range
floating slash  format  allows scaled  up  numerators  (implicit

denominator  unity)   and  scaled   up  denominators  (implicit
numerator  unity)  to  achieve  ranges  comparable  to  typical
floating point  ranges. This  provides a  reasonably  convenient
solution to the range problem for rational representation.
    Precision  problem:   We  wish  to point  out  a  "precision
fill" feature can  be created in  floating slash  representation
to achieve reasonably uniform relative spacing. This is possible
by treating  the  presence  of  a  non unit  valued  GCD ·as  an
indication that more accuracy is to be provided by  interpreting
the value  of  the GCD  as· giving leading bit  information  on
the next partial . quotient in the  expansion of the  approximate
value.I.e. 1000/2000  in  such a  "denormalized"  interpretation
would have  the GCD  of  1000 indicate a small  deviation· from
1/2 whose  precise meaning requires  more discussion. We  simply
summarize here that the current redundancy of a simple  fraction
(GCD values  available)  for  each fraction  in  floating  slash
is of  just  the  right order  to  appropriately  allow  filling
the gaps  on both  sides of  the  simple  rational. "Appropriate"

means that n  bit floating slash can  achieve accuracy of  about
say one  part in  2^(n-k)  for k  a constant  bounded by  two  or
three in an appropriate  development of this feature. This  idea
is currently  under investigation by  these authors  as part  of
further developments  in  our investigation  of  floating  slash
arithmetic.

3.An  alternative continued  fraction based  rational system  is
also available

The use of  a scaled  version of k-bit  LCF bitstrings  provides
an alternative  to  floating slash  representation with  a  very
similar  and  competitive  arithmetic  unit  architecture. Such
systems have far less gap variation between k-bit  representable
values than  in  standard floating  slash  and still  allow  for
exact  representation  of  all  appropriately  simple  rational
values.We  suggest that  scaling  to achieve  a  large  range
is a  reasonably  achievable  extention to  the  unscaled  k-bit
LCF format. The  unscaled LCF  representation and  corresponding
arithmetic unit are described in the following two references:

> Kornerup  ,  P.  and Matula,   D.W.:"Finite  Precision
> Lexicographic Continued Fraction Number Systems", Proc.
> 7th Sym. on  Comp. Arith., IEEE  Cat #85CH2146-9, 1985,
> 207-214.

> "An On-Line Arithmetic  Unit for Bit-Pipelined Rational
> Arithmetic",  J.  Parallel  and Distributed  Comp.,  5,
> 1988, 310-330.

4.Further references.

For further  information  on floating  slash  representation  we
note the following two references:

> Kornerup,   P.  and Matula,   D.W.: "Finite  Precision
> Rational Arithmetic: An Arithmetic Unit", IEEE Trns. on
> Comp., C-32, 1983, 378-388.
> Matula,   D.W.  and Kornerup,   P.: "Finite  Precision
> Rational Arithmetic: Slash Number Systems", IEEE Trans.
> on Comp., 1985, 3-18.

David W. Matula
Peter Kornerup
Aarhus, 22 July 1988