WHAT'S WRONG WITH THESE PROGRAMS ?

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Imagine encountering these problems on an exam.

1.

```
Real  x, y, z ;
Real  E(z) := if  z = 0  then  1
                         else  (exp(z) - 1)/z ;
Real  Q(y) := (SQRT(y²+1) - y) - 1/(SQRT(y²+1) + y) ;
Real  G(x) := E(Q(x)²) ;
For integer  n = 15 to 100  do  Display { n, G(n) } .
```

If this program were carried out using exact arithmetic, with no roundoff error, the displayed values of G(n) would all be 1 . But that will not happen.

Translate this program into a suitable language and run it on any computer you like that performs real arithmetic approximately. As long as every arithmetic operation is rounded (or chopped) to the nearest rational number representable with the same previously chosen finite number of significant digits (as happens in almost all computers' floating-point arithmetics), no matter how many digits, almost all the displayed values will be the same wrong number. What number? And why?

2. This program is due to Jean-Michel Muller:

$A_0 := 11/2$; $A_1 := 61/11$;
For $n = 1$ to 39 do $A_{n+1} := 111 - (1130 - 3000/A_{n-1})/A_n$;
Display A_{40} .

If this program were executed with exact rational arithmetic, no roundoff, the computed values would be $A_n = 6 - 1/(1 + (6/5)^n)$, so $A_{40} = 5.99932...$. But that will not happen if any floating-point arithmetics built into the hardware of computers are used instead; they all get the same wrong number for A_{40} . What is it? And why?

3. This Fortran expression was used by James Sethian :

```
F(X, Y)  =  ACOS( X/SQRT( X*X + Y*Y ) )
```

Assume 0.0 < X*X + Y*Y < Infinity and no over/underflows. How can F(X, Y) go wrong? For some eligible arguments X and Y it aborted on a CRAY because roundoff caused the argument of ACOS to exceed 1.0 . This has not happened yet on any other computer or calculator. Can you tell whether eligible arguments X and Y exist on your computer that will abort F(X, Y) ? Should the expression for F(X, Y) be changed? If so, to what?